Flight Tests Demonstrate Sub 50 cms RMS Vertical WADGPS Positioning

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BIOGRAPHY

Ronald Muellerschoen received a B.S. degree in physics at Rensselaer Polytechnic Institute and a M.S. degree in applied math at the University of Southern California. He is currently a Member of the Technical Staff in the Orbiter and Radiometric Systems Group at the Jet Propulsion Laboratory (JPL). His work at JPL has concentrated on the development of filtering software for processing GPS data, real-time GPS data processing, and development of wide area differential systems.

Willy Bertiger received his Ph.D. in Mathematics from the University of California, Berkeley, in 1976. In 1985, he began work at JPL as a Member of the Technical Staff in the Orbiter and Radiometric Systems Group. His work at JPL has been focused on the use of GPS, including high precision orbit determination, positioning, geodesy, remote sensing, and wide area differential systems.

Michael Whitehead received his Ph.D. in Electrical Engineering from the University of Florida in 1986. He developed the software that serves as the backbone for Satloc's wide-area differential system. This includes clock-filter algorithms, communications, integrity, and user interfaces. Most of Mike's past work experience has involved a combination of software, signal processing, and control.

ABSTRACT

Wide Area Differential GPS (WADGPS) positioning is performed in real-time during NASA's DC-8 AirSAR flights. The goal of the experiment is to demonstrate absolute positioning in earth-fixed coordinates to better than one meter in all components in real-time. Results show dual-frequency real-time RMS (root-mean-square) accuracy in the vertical to be 50-60 cms with an RMS horizontal accuracy of better than 40 cms.

Augmenting the system with stable Rubidium oscillators both at a reference ground receiver and on the aircraft allows the user to model his clock as a predictable process rather than a stochastic white-noise process. This permits better separation of the clock and vertical estimates. Results of this experiment demonstrate dual-frequency real-time RMS accuracy in the vertical to be better than 40 cms RMS.

An important aspect to real-time GPS positioning experiments is verifying accuracy. Post-processing techniques using only GPS data are shown to yield truth solutions with an accuracy in all components of better than 10 cms RMS.

SATLOC'S WIDE-AREA DIFFERENTIAL SYSTEM

Since May of 1997[+], Satloc has been providing Wide Area Differential GPS (WADGPS) correction services to commercial users over the continental United States (CONUS) and parts of Canada and Mexico. Core software processes of their system were developed at JPL. These codes additionally serve as a prototype for the Federal Aviation Administration's (FAA) Wide Area Augmentation System (WAAS) [1] being brought to

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[+] In April of 1999, OmniSTAR purchased the network from Satloc.

development by Raytheon [2]. These processes provide real-time estimates of the ionosphere and dynamic GPS orbits, and one-second GPS clock corrections. Corrections are only needed every 5 minutes for the GPS orbits and ionosphere due to their slower varying physical behavior.

The source of Satloc's corrections is a network of 15 dual-frequency Ashtech Z-12 GPS receivers. One-second GPS data is transmitted to two redundant network control centers (NCCs) over frame-relay communication links using TCP/IP protocol. One NCC is located in Reston VA, while the other is in Scottsdale, AZ. Additionally the data stream is also transmitted to JPL for purposes of development and debugging. At the NCCs a network of Pentium PCs running Windows NT processes the data and computes the WADGPS corrections. References [3] and [4] contain details of the core algorithms.

These corrections are packaged into a 750 bps data message and then viterbi encoded to a 1500 bps data stream. This is uplinked to the American Mobile Satellite Corporation's (AMSC) L-Band communications satellite where is it is broadcasted in 3 overlapping spot beams over the United States and parts of southern Canada and northern Mexico. Reference [5] contains additional details of Satloc's NCC operations, network infrastructure, integrity software, and steps taken to achieve 99.997% availability of their WADGPS signal. In addition, Reference [6] sites Satloc's superior availability in kinematic tests versus other competing satellite-based commercial systems, including WAAS.

Some of the noteworthy differences between Satloc's WADGPS system and FAA's WAAS are 1.) network latency is less than 4 seconds versus 6 seconds for WAAS, 2.) clock updates are at 1 second versus ~6 seconds for WAAS, 3.) 15 stations are used in the Satloc network and only in CONUS whereas WAAS has a redundant network of 24+ receivers including Alaska and Hawaii, 4.) to meet stringent availability requirements due to high integrity prerequisites, WAAS requires two or more Geostationary satellites to additionally transmit GPS-like ranging tones, 5.) the WAAS correction signal is 250 bps versus 750 bps for Satloc's signal, both before viterbi encoding, and 6.) the WAAS corrections are quantized to 1/8 of a meter versus 1/16 of a meter for Satloc's corrections.

JPL'S RTG AND RTI SOFTWARE

The software producing the corrections at the Satloc's NCCs is licensed from JPL. The GPS orbits and the one-second clock corrections are computed with JPL's Real-Time Gipsy (RTG) software. The ionosphere vertical delays are computed with JPL's Real-Time Ionosphere (RTI) software. For redundancy, Satloc developed an alternate one-second clock process. Common failure modes between the two clock processes are unlikely because of the independent implementations. Automatic rollover occurs should the chosen primary process fail.

For the user, Satloc integrated a single-frequency GPS receiver with an L-Band receiver. The L-Band receiver processes the WADGPS correction signal and converts it to RCTM Type 1 SC-104. The GPS receiver uses this differential correction to adjust its measurements to account for errors in the broadcast GPS orbit and clock, and errors due to the ionosphere.

On the JPL's user side, the RTG and RTI software sets are hosted on a PC running Linux. Satloc's WADGPS corrections transmitted by the Geostationary satellite can be directly combined in software with the measurements from either a dual-frequency Ashtech Z-12 or AOA Turbo-Rogue GPS receiver, or a single-frequency Ashtech G-12, to compute the user's position and clock.

For NASA's DC-8 AirSAR flights, there is a dual-frequency Ashtech Z-12 GPS receiver along with an L-Band receiver to acquire Satloc's WADGPS corrections. The receivers are attached to the serial ports of a Toshiba Tecra PC where the data streams are combined with RTG to compute the phase center of the GPS antenna located on top of the aircraft.

Extensive use of shared memory and signals are used to communicate among the different RTG modules hosted on the PC. For example, a GPS data daemon reads in the byte stream from the GPS receiver and places the observables into a circular shared memory buffer. Other processes can read the same data to either write rinex files for subsequent post-processing, or to compute the user's position and clock with various options. Multiple user positioning's can be performed simultaneously, such as dual-frequency versus single-frequency, or with varying GPS elevation cutoffs of the data. Being able to run different user positioning processes on the same data sets in real-time allows for quick debugging and tuning of the algorithms.

NON REAL-TIME TRUTH SOLUTIONS

By post-processing the rinex files captured from the PC's shared memory, a truth solution can be produced to assess the accuracy of the real-time solutions. For post-processing, JPL's GIPSY OASIS II software (GOA II) [7][8] and precise orbits[+] and one-second clocks are used to reduce the data. GOA II has a long history in precise orbit determination of GPS and other spacecraft equipped with GPS receivers [10][11][12][13][14], and in precise GPS geodetic applications [15][16].

The high accuracy obtainable in the post-processed truth solutions is the result of 1.) using precise GPS orbits accurate to better than 20 cms RMS, and precise GPS clocks accurate to a few-tenths of a ns, 2.) robust data-editing algorithms [17], and 3.) iteration and smoothing of the kinematic filtered solution.

On October 16, 1998 AirSAR flight, GPS data was collected onboard the DC-8 while simultaneously members of JPL's AirSAR group collected 2 hours of GPS data on the ground at Rosamond, CA near Edward's A.F.B. (Edward's serves as the staging area for the AirSAR flights.) Ground data was collected with an Ashtech Z-12 receiver, identical to one used on the DC-8. The purpose of this data was to detect any interference the AirSAR L-Band radar pulses might have on a typical GPS user. The test proved negative; no interference was detected. Moreover, the GPS data collected at Rosamond provided an opportune data set to access the accuracy of the post-processed aircraft truth solutions.

The strategy to establish the accuracy of the inflight kinematic truth solution of the aircraft is as follows: First the truth position of the ground antenna at Rosamond, CA is determined by post-processing with a stationary assumption and using precise GPS orbits and precise 300-second GPS clock solutions. Next, the location of the ground antenna is solved for kinematically at a one-second rate, using the same method and same orbit and clock database that is used to compute the inflight kinematic truth solution of the aircraft. The scatter in the

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[+] The orbits and 300-second clocks used in the truth processing are JPL's rapid-orbit submission to the International GPS Service (IGS) [9]. The orbits differ less than 20 cms RMS with the final IGS combined solution.

kinematic solution of the ground antenna as compared to its static solution should then represent the precision of the inflight truth solution of the aircraft, notwithstanding multipath differences between the two environments. To determine whether aircraft multipath is significant in the inflight truth solution, ambiguity resolution is performed between the kinematic solution of the ground antenna and the inflight kinematic truth solution of the aircraft. If certain conditions are met, then the accuracy of the inflight truth solution should be within the scatter that results by differencing the inflight truth solution with the inflight ambiguity-resolved solution. These conditions are that ambiguity resolution yields no degradation in the scatter of the kinematic ground solution, and that the difference between the inflight truth solution and the inflight ambiguity-resolved solution is reasonably small.

Details of the Static Truth Solution

As mentioned, an accurate static solution for the ground antenna is needed to judge the accuracy of its kinematic solution. Due to logistics, only 2 hours of data is available to solve for the location of the ground antenna at Rosamond. It is well known that with 24 hours of dual-frequency GPS data, the position of a static antenna can be determined to within 1-2 cms [18]. However with only 2 hours of data, it is not clear what level of accuracy can be expected.

To answer this question, 2 days of data from an Ashtech Z-12 receiver in Arcata, CA was processed with X hours of data in each arc, at X hour increments. Table 1 lists the repeatability of the coordinates relative to its known location. From Table 1, 2 hours of data is sufficient for 3 cm accuracy in horizontal and 4 cm accuracy in vertical. Note that these are "1-sigma" numbers.

Details of the Kinematic Truth Solutions

To compute the kinematic solution of the ground antenna, precise orbits and one-second data from Satloc's ground network are first used to compute precise one-second GPS clocks. Holding both the orbits and the solved-for one-second clocks fixed, the one-second ground data from the GPS receiver at Rosamond is passed through GOA II, kinematically estimating its position and stochastic clock. The antenna's zenith troposphere delay is modeled as an exponential function of geodetic height. It is not estimated since it would correlate strongly with the vertical component of position. This is in contrast to the static positioning of the data where the zenith troposphere

delay is estimated as a constrained random-walk. Table 2 summarizes the differences between the kinematic solution and the static solution.

Table 1.   RMS repeatability for X hours of data from a dual-frequency static GPS receiver.

X	East (cm)	North (cm)	Vertical (cm)
1 hour	4.3	2.8	7.2
2 hours	2.8	1.3	3.7
3 hours	2.0	0.6	2.7
4 hours	1.2	0.6	1.6

The mismodeled and un-estimated troposphere delay in the kinematic solutions is likely the cause of the -7.1 cm mean difference with the vertical of the static solution. In fact when the solved-for troposphere delay from the static ground solution is used and held fixed in the kinematic solution, the mean in the vertical decreases to -3.9 cms. The substantial 7.8 mean in the East component may be due to using 300-second clocks in the static case versus using the solved-for 1-second clocks with the network data in the kinematic case. When true IGS quality orbits and 300-second clocks are used in the static solution, the mean in the East component decreases to 5.8 cms.

Table 2. 2-hours of one-second kinematic position estimates of the dual-frequency GPS receiver at Rosamond, CA. Difference is with the static solution.

	mean (cm)	sigma (cm)	RMS (cm)
East	7.8	1.7	8.0
North	2.5	1.2	2.7
Vertical	-7.1	2.8	7.6

The same method of kinematically estimating the GPS antenna position at Rosamond is applied to estimating the position of the GPS antenna on the DC-8. However, one difference that needs to be accounted for is the antenna

phase wind-up [19] of the mobile antenna on the DC-8. In this case, the x-component of the effective dipole is chosen to lie along the Earth-fixed velocity vector of the aircraft. The normal to the dipole plane is in the direction of the velocity crossed with the aircraft's cross-track direction. For most typical aircraft attitudes, this is a well defined reference frame.

If the zenith troposphere delay model has similar accuracy for all altitudes, then the accuracy seen in the kinematic Rosamond solution should then be the accuracy of the inflight kinematic truth solution of the aircraft. From Table 2 it is clear that the accuracy in the truth solutions should be less than 10 cms RMS in all components.

Building Confidence with Ambiguity Resolution

As an additional measure of confidence in the truth accuracy, the double-differenced phase ambiguities between the GPS antenna on the aircraft and the GPS antenna at Rosamond are resolved. The method from Reference [20] is used in which first the wide-lane combinations are first resolved with a geometry-free approach, and then the narrow-lane combinations are resolved after applying a precise model to the observables. Table 3 lists all 18 of the non-redundant double-differenced phase ambiguities for the two-hour data set.

The Ashtech Z-12 provides high quality code observables, and since the wide-lane wavelength is 86.2 cms, all the wide-lane ambiguities are easily resolved to their nearest integer. Deciding whether a narrow-lane can be resolved is more difficult since the narrow-lane wavelength is only 10.7 cms. Additional information such as the formal error of the double-difference estimate, in addition to the difference of the number from it's nearest integer, is taken into account. A bootstrapping technique is used to first resolve the obvious narrow-lane ambiguities. The positions are then recomputed with the resolved ambiguities in place, and additional narrow-lane ambiguity resolution is attempted. In the Rosamond/aircraft data, as table 3 indicates, 3 iterations are required to safely resolve all the double-differenced ambiguities.

Table 3. Double-differenced phase ambiguities between the aircraft and GPS receiver at Rosamond, CA. Integer nature of the numbers indicate that the phase ambiguities can be fixed to the nearest integer. After the 3rd pass, the remaining two computed narrow-lanes are fixed to their nearest integers.

wide-lane (N1-N2)	computed or fixed narrow-lane (N1)		
	1st pass	2nd pass	3rd pass
-4.916	19.932	fixed at 20	fixed at 20
-3.943	-0.672	-0.801	fixed at -1
-14.046	40.485	40.412	40.144
5.016	-22.632	-22.801	fixed at -23
-9.047	21.960	fixed at 22	fixed at 22
-19.968	55.954	fixed at 56	fixed at 56
-15.006	33.321	33.198	fixed at 33
24.993	34.006	fixed at 34	fixed at 34
-19.991	11.373	11.198	fixed at 11
-10.995	-12.807	fixed at -13	fixed at -13
-21.992	53.450	53.198	fixed at 53
-26.989	76.082	fixed at 76	fixed at 76
1.994	11.470	11.198	fixed at 11
-2.975	34.103	fixed at 34	fixed at 34
-24.026	59.267	59.198	fixed at 59
-28.968	81.900	fixed at 82	fixed at 82
-0.998	0.757	0.906	fixed at 1
2.912	10.679	10.245	9.877

The results of fixing all the double-differenced phase ambiguities and re-computing the kinematic position of the antenna at Rosamond is summarized in Table 4. In terms of scatter of the position estimate, there is significant improvement in the East component. The improvement in the vertical is inconsequential since the vertical errors are relatively uncorrelated with the double-differenced phase ambiguities. The non-zero mean in the East component may in part be due to the unreliable static estimation of the antenna's actual position as mentioned above. Note however that these means are still within the expected "3-sigma" error that a two-hour static data set can be expected to provide. The mean in the vertical is likely due to modeling the zenith troposphere delay as an exponential model of the geodetic height instead of estimating it as a constrained random-walk parameter as in the static case.

Table 4. 2-hours of one-second kinematic position estimates at Rosamond, CA after ambiguity resolution with the aircraft data.

	mean (cm)	sigma (cm)	RMS (cm)
East	8.0	0.7	8.0
North	0.0	1.2	1.2
Vertical	-5.4	1.9	5.7

Finally, Table 5 summarizes the differences between the aircraft truth solution and of the aircraft solution with all phase ambiguities resolved. Note that the aircraft truth solution here refers to the unresolved solution. It can be argued that the real truth is where all the ambiguities have been resolved. But to determine the accuracy of the truth solution, a better truth is needed to compare with. More typically there is not a secondary receiver in the vicinity of the aircraft that can be used for ambiguity resolution.

From Table 2 it is argued that the aircraft truth solutions should yield horizontal accuracy better that 10 cms RMS. The 7.8 cm mean in the East component and the 8.0 cm mean in East from Table 4, suggest that the statically determined position of the antenna may be in error. From Table 5, the horizontal truth accuracy may even be better than 10 cms RMS.

Table 5. Difference between the aircraft solution with and without all double-differenced phase ambiguities resolved.

	mean (cm)	sigma (cm)	RMS (cm)
East	2.1	2.9	3.6
North	3.4	1.0	3.5
Vertical	-6.0	4.0	7.2

When all phase ambiguities between the ground and the aircraft GPS antennas are resolved, improvement is seen in the scatter of the kinematic ground solution. Consequently, the accuracy of the horizontal components of the aircraft truth solution should be the scatter that results by differencing the truth solution with the ambiguity-resolved solution. From Table 5 then, it is likely that the horizontal components are better than 4 cms RMS.
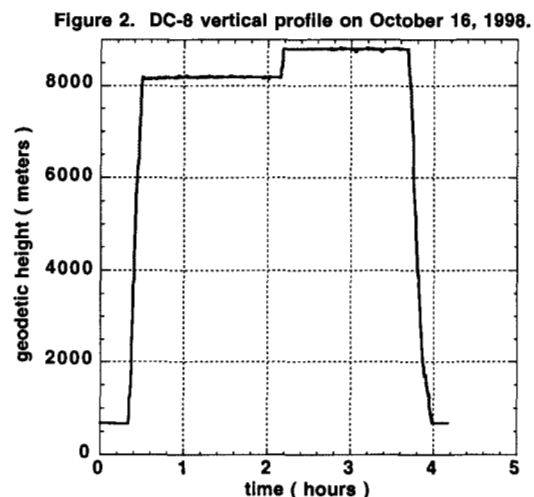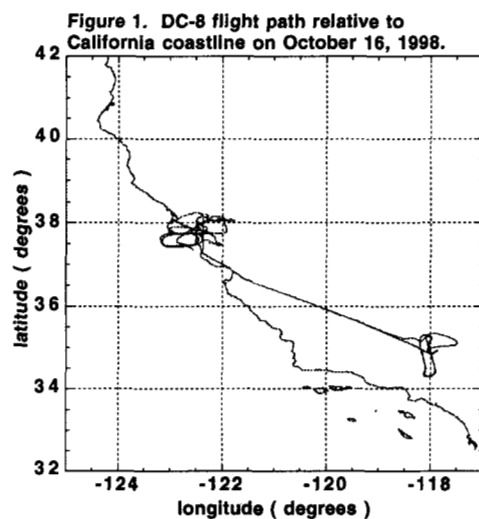
The same can not be said of the vertical component since it is not very sensitive to ambiguity resolution. The dominant error in the vertical is the modeled exponential zenith troposphere delay. At least at ground-level, from Table 2, the vertical component is accurate to better than 8 cms RMS. At a flight level of 8.8 kms, the exponential model of the zenith troposphere delay may be off as much as 10 cms (see appendix). Some of this mismodeled troposphere delay is absorbed by the user's clock, and some translates into a vertical bias of aircraft solution. Pessimistically the expected accuracy of the aircraft's truth vertical component is greater than 10 cms RMS, but most likely less than 20 cms RMS.
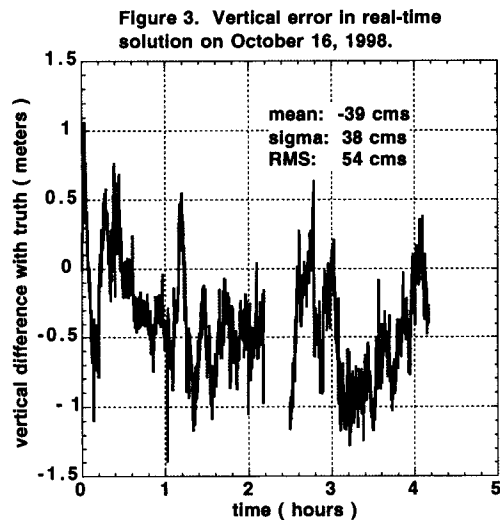
Later AirSAR flights in May and June of 1999, briefly discussed at the end of this paper, use the external air pressure as input to better model the zenith troposphere delay in both the real-time and post-processed solutions. In these cases, the troposphere model is no longer the dominant error. So again from Table 2, if the external air pressure is known, the vertical component of the truth solutions should be accurate to better than 8 cms RMS.

ACCURACY OF REAL-TIME SOLUTION

Figures 1 and 2 show the 4+ hour flight path of the DC-8 on October 16, 1998. The 680 meter altitude at the beginning and at the end of Figure 2 represents the altitude of the runway at Edward's A.F.B. Figure 3 shows the vertical difference of the real-time solution with the post-processed truth solution to be 54 cms RMS. There is a 20 minute gap in Figure 3 between hours 2 and 3. During this time the GPS pre-amp was connected to a spectrum analyzer to determine if the L-Band pulses transmitted by the radar could be detected by the GPS antenna on top of the aircraft. The last 2 hours which include the racetracks over Rosamond, CA are used in the above discussed ambiguity resolution processing. Additionally, Table 6 demonstrates the horizontal accuracy of the real-time solutions to be from 30 to 40 cm RMS.
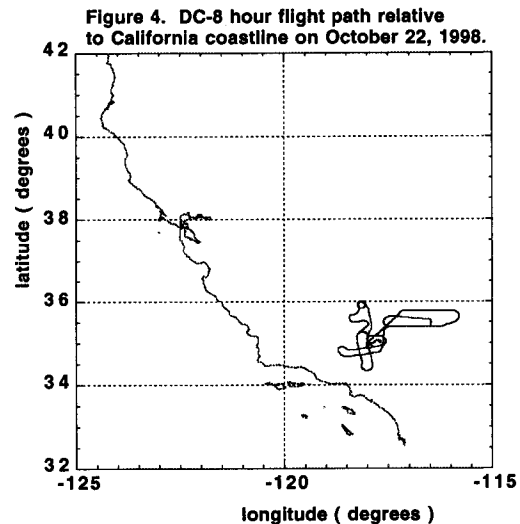

Figure 1. DC-8 flight path relative to California coastline on October 16, 1998.

Figure 2. DC-8 vertical profile on October 16, 1998.

Figure 3. Vertical error in real-time solution on October 16, 1998.

mean: -39 cms
sigma: 38 cms
RMS: 54 cms

Figure 4. DC-8 hour flight path relative to California coastline on October 22, 1998.

Table 6. Difference between the real-time aircraft solution and the truth aircraft solution on October 16, 1998.

	mean (cm)	sigma (cm)	RMS (cm)
East	18	22	28
North	-29	31	43
Vertical	-39	38	54

Table 7. RMS difference between the real-time aircraft solutions and the truth aircraft solution on October 22, 1998.

	dual freq.	single freq.
East	15 cms	30 cms
North	35 cms	49 cms
Vertical	47 cms	124 cms

Reference [21] also briefly discusses the results of an earlier AirSAR flight on June 4, 1998. On this day, the real-time vertical position error is 63 cm RMS. Both East and North components are less than 30 cm RMS.

Figure 4 shows the 3.5 hour flight path of the DC-8 on October 22, 1998. Table 7 lists the RMS difference between the real-time and truth solutions. In addition to the real-time dual-frequency solution, a real-time single-frequency solution is also computed. In Figure 5, every second of the 3.5 hour flight is represented. There are no missed epochs and no outliers present. The top line of Figure 5 is the 124 cm RMS vertical error of the single-frequency solution and the bottom line is the 47 cm RMS vertical error of the dual-frequency solution.

Figure 5. Vertical error in real-time solution on October 22, 1998.

dual freq. (bottom line)
single freq. (top line)

SUB 40 CMS RMS VERTICAL ACCURACY WITH STABILIZED CLOCKS

A receiver's clock is typically modeled as a stochastic white-noise process. There is a substantial 96% correlation between the clock and vertical estimates since the partials for both are nearly identical, particularly for high elevation data. If however the user's clock is driven with a stable frequency standard, then its clock can be modeled instead as a predictable process, such as a low-order polynomial. This allows for separation of the clock and vertical estimates as they decorrelate over time. Moreover, if the user's WADGPS clock solution is relative to the WADGPS reference clock, the reference clock too must be driven by a stable frequency standard.

Before the AirSAR flight of November 13, 1998, Satloc attached a Rubidium frequency standard to their Ashtech Z-12 receiver in LaJollo, CA. This receiver, and a backup receiver, are typically steered by an Hewlett-Packard ovenized crystal oscillator to maintain a near-zero clock rate relative to GPS time. This is necessary so that the available dynamic range of the WADGPS corrections relative to the broadcast clock is not saturated.

Figure 6 shows the precise clock solution at LaJollo, CA relative to a ultra-stable hydrogen maser at Goldstone, CA. Represented in this plot is both before and after the installation of the Rubidium frequency standard. There is brief settling period at turn-on of the oscillator.
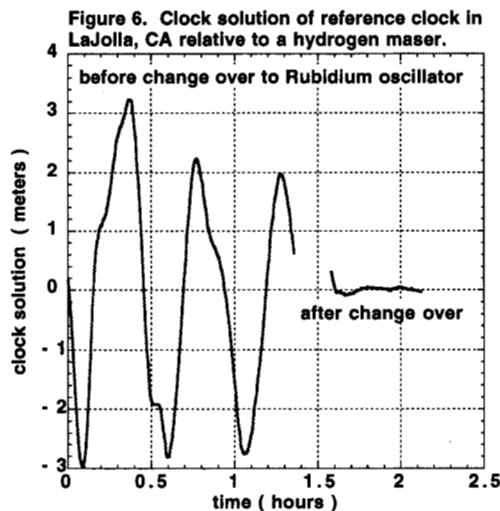
Figure 6. Clock solution of reference clock in LaJolla, CA relative to a hydrogen maser.

Figure 7 show the stability of the clocks in terms of an Allan variance. The bottom line in this plot represents the stability of another hydrogen maser clock at North Liberty, Iowa. With the Rubidium oscillators, the clocks at the WADGPS reference station and onboard the DC-8 are stable to a few parts per 10^{13} for periods on the order of several tens of minutes.

Figure 7. Clock stabitily of GPS receivers relative to a hydrogen maser.
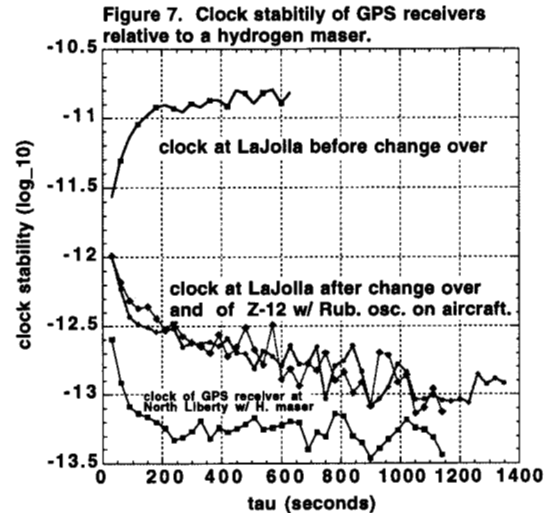
With the stable clocks in the system, multiple real-time positionings with a white-noise clock model and quadratic clock model are performed. Table 8 shows the difference between the real-time aircraft solutions and the truth aircraft solution. So as to not over-determine the coefficients of the quadratic clock model, a small amount of process noise is added to the quadratic term of the model at each epoch. Results from this table show sub 40 cm RMS vertical accuracy.

Table 8. RMS difference between the real-time aircraft solutions and the truth aircraft solution on November 13, 1998.

	white-noise clock model	quadratic clock model
East	27 cms	33 cms
North	32 cms	26 cms
Vertical	61 cms	39 cms

Figure 8 represents a short 20-minute segment of vertical errors in the November 13, 1998 solution. The smoother line in this plot represents the positioning with the quadratic clock model. From this plot it is evident that in addition to improving accuracy, adding stable clocks to the system also increases the robustness and smoothness of the estimates.
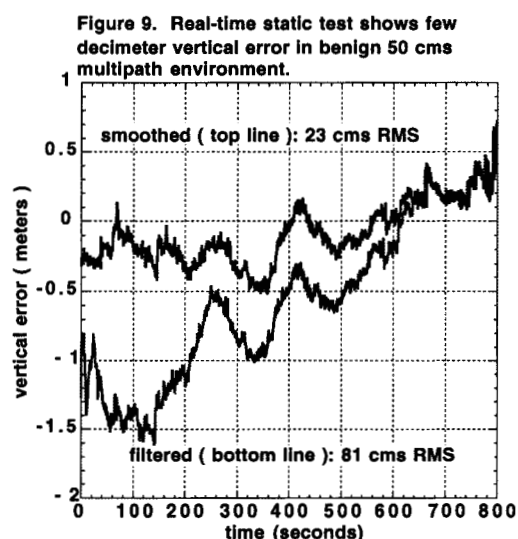
Figure 8. Vertical position errors with quadratic versus white-noise clock model from 18:40 to 19:00 November 13, 1998.
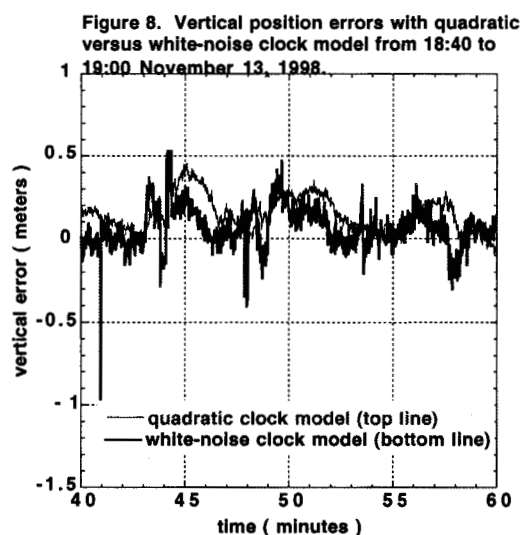
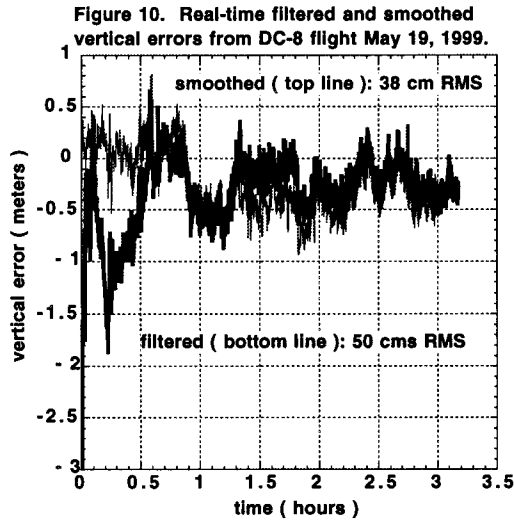STATIC GROUND RESULTS, MULTIPATH, AND SMOOTHING

Reference [4] demonstrates real-time steady-state few decimeter position accuracy of static dual-frequency receivers using Satloc's WADGPS corrections. Furthermore, Figure 9 shows both real-time filtered and smoothed WADGPS solutions for a static receiver from start-up to 13+ minutes. As evident in this plot, a few minutes is necessary for convergence of the estimated carrier phase biases. The smooth solution uses the terminal solved-for carrier phase biases to better estimate the position of antenna at the beginning of the arc. The few decimeter steady-state accuracy demonstrated in Reference [4], is also represented in the smoothed solution of Figure 9. In all these static cases, there is a benign multipath environment of 50-100 cms. Multipath here is computed as taking the standard-of-deviation of the difference between the one-second ionosphere-free linear combination of the range data minus the one-second ionosphere-free linear combination of the phase data:

$$2.54 \, P1 - 1.54 \, P2 - (2.54 \, L1 - 1.54 \, L2)$$

On the DC-8 aircraft, multipath is typically 2-3 times worse than these static cases. Consequently the positioning results are expected to be 2-3 times worse until the phase bias estimates have converged. The position accuracy is particularly worse at the beginning of a data arc. Figure 10 shows both real-time filtered and smoothed vertical errors from a recent May 19, 1999 AirSAR flight. The initial non-steady state vertical accuracy of the DC-8 filtered solution is in fact about 2-3 times worse than the static results, and it requires about 30+ minutes to reach convergence to the smoothed solution. Since phase breaks are detected continuously and may occur frequently in an aircraft environment, the filtered and smoothed solutions have better agreement towards the end of the data arc; by definition of course they agree exactly at the end of the arc.

Although smoothing can not be performed in real-time, the RTG smoother also hosted on the PC running Linux, requires only a few seconds after the real-time filtered solution is completed to compute a 3+ hour smoothed solution. For some applications, such as processing SAR data immediately upon landing, smoothing the real-time filtered solution can add significant accuracy without waiting for additional data such as precise GPS orbits and clocks.

Figure 9. Real-time static test shows few decimeter vertical error in benign 50 cms multipath environment.

Figure 10. Real-time filtered and smoothed vertical errors from DC-8 flight May 19, 1999.

smoothed (top line): 38 cm RMS

filtered (bottom line): 50 cms RMS

Stabilizing the clocks in the system with Rubidium oscillators allows for better separation of the vertical and clock components. Results of this experiment demonstrates dual-frequency real-time RMS accuracy in the vertical to better than 40 cms RMS.

APPENDIX

In RTG and GOA II, the zenith troposphere delays can be modeled as exponential functions of geodetic height (h).

$$W(h) = 0.1 \exp^{(\frac{-h}{2000})}$$

$$D(h) = 2.29951 \exp^{(\frac{-0.116\ h}{1000})}$$

SUMMARY

Accurate real-time position solutions have been produced onboard NASA's DC-8 aircraft using differential corrections transmitted through a Geostationary satellite. Much of the software used in generating these corrections is licensed to Satloc by JPL. The differential L-Band receiver hardware was purchased from Satloc. Other hardware required for this demonstration is a dual-frequency Ashtech Z-12 GPS receiver for collecting observables and a laptop computer. The laptop performs real-time positioning using JPL's Real-Time Gipsy (RTG) software.

Inflight kinematic truth solutions of the aircraft are computed by post-processing the flight data with precise orbits and precise one-second GPS clocks. The horizontal accuracy of these truth solutions are better than 4 cms RMS. The vertical accuracy of these truth solutions are better than 8 cms RMS if the external air pressure is known, since it can be used to better model the dry zenith troposphere delay. If the external air pressure is not known, then the vertical accuracy of the truth solution may be as worse as 20 cms RMS.

The accuracy of the real-time solutions can be determined by differencing them with post-processed truth solutions. Results show dual-frequency real-time RMS accuracy in the vertical to be 50-60 cms and an RMS horizontal accuracy of better than 40 cms.

where h is the geodetic height in meters and W(h) and D(h) are respectively the wet and dry zenith troposphere delays, also in meters. At typical aircraft altitudes, the wet zenith delay is sub cm and can therefore be ignored. Moreover, if the external pressure is known, a more accurate model of the dry zenith delay can be computed by applying hydrostatic equilibrium equations to a column of dry air. From the ideal gas law:

$$PV = n\,R\,T$$

where P is pressure, V is volume, n is the number of moles, R is the ideal gas constant 8.31432 joules / degrees K / mole, and T is temperature in degrees Kelvin. Multiplying both sides by the molecular weight of dry air M (0.028964 kg/mole) and dividing by the volume, the density of air can be expressed as:

$$\rho = \frac{M\,P}{R\,T} = 0.003484\,\frac{P}{T}$$

The dry component of the zenith atmospheric delay can be computed by

$$D = -\int_{h}^{\infty} (n-1)\,ds$$

where the refractivity of dry air is given by

$$N = (n-1) \times 10^6 = 0.7757 \frac{P}{T} = 222.65 \, \rho$$

The hydrostatic equilibrium equation states that

$$P = -g \int_h^\infty \rho \, ds$$

where g is the acceleration of gravity. Substituting yields:

$$D = \frac{P}{g} \, 222.65 \times 10^{-6}$$

$$D = 2.27 \times 10^{-5} \, P \, (pascal) = 0.00227 \, P(mbar)$$

D is now simply a function of the external pressure in millibars; its units are meters. At an aircraft altitude of 8.8 kms, the atmospheric pressure outside the aircraft is typically 320 mbar[+]. The difference then between this hydrostatic model and the exponential geodetic height model at this altitude is 10 cms.

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[+] This number is based on flight information noted on later AirSAR flights in May and June of 1999. It is not known what the external pressure at 8.8 kms altitude was during the October 16, 1998 flight.

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